Chapter 47
An Improved Algorithm for Single Point Positioning of COMPASS

Guorui Xiao, Lifen Sui, Bing Wang, Yu Gan, Qinghua Zhang, Yu Duan and Guobin Qi

Abstract The BeiDou (COMPASS) Navigation Satellite System has been able to provide service for areas in China and its surrounding areas. However, due to the limited number of satellites and the low accuracy, its application is greatly limited. The global nonlinear least squares algorithm (Bancroft algorithm) is applied in COMPASS single point position in this paper. The Bancroft algorithm based on the Lorentz inner product is mainly in four-dimensional space. In consideration of the constellation of COMPASS is composed of Geostationary Earth Orbit (GEO) satellites and Non-Geostationary Earth Orbit (Non-GEO) satellites, A new method to get the COMPASS observation weights is introduced base on the Lorentz inner product. The observation data is computed to demonstrate the ideas involved. The results indicates that the method can improve the accuracy of COMPASS single point positioning.

Keywords Bancroft · Single point positioning · COMPASS · Weighting algorithm · Nonlinear least squares

47.1 Introduction

In global positioning system applications several error sources affect the observations. Tropospheric delay that occurs during the propagation of the wave through the troposphere and multipath taking place as a result of signal reflection are the most important error sources [1]. One way to overcome this problem is to introduce of new

G. Xiao (✉) · L. Sui · B. Wang · Y. Gan · Q. Zhang · Y. Duan · G. Qi
Geospatial Information Institute, Information Engineer University,
Zhengzhou 450052, China
e-mail: xgr@whu.edu.cn

J. Sun et al. (eds.), *China Satellite Navigation Conference (CSNC) 2013 Proceedings*, Lecture Notes in Electrical Engineering 243,
DOI: 10.1007/978-3-642-37398-5_47,

stochastic models. Ward [2] gives the $Sigma - \varepsilon$ model as a function of the measured C/N_0 values. In addition to this stochastic model, another model based on elevation cut-off angles has been applied by Rothacher [3]. Both models mentioned above are applied to GPS data at zero difference level, where the observations are undifferenced. The second model was implemented into the 4.2 version of Bernese GPS processing software. The constellation of GPS is composed of 24 MEO satellites located 20,231 km height in space while the constellation of COMPASS is composed of Geostationary Earth Orbit (GEO) satellites and Non-Geostationary Earth Orbit (Non-GEO) satellites [4]. The weighting algorithms used in GPS is not applicable in COMPASS because the satellites height are different.

For GPS single point positioning, the local area optimal solution is solved by the iteration of linearized least square [5]. But it may get different results and even can make the result incorrect because of the different approximate coordinates in the linearized process or the different receivers' positions (such as the receiver is not on the ground of the earth). Thus the American scholar Steven Bancroft [6] puts forward a global nonlinear least squares algorithm called the "closed-form" solution for GPS pseudorange equations in 1985. This algorithm does not need linearization, and it is one of nonlinear solution methods having the algebraic and analytic noniterative characteristics [7–9].

Global nonlinear least squares algorithm (Bancroft algorithm) is applied to COMPASS single point positioning in this paper and a new weighting algorithm is introduced base on the Lorentz inner product in Bancroft algorithm. The observation data is computed to demonstrate the ideas involved. The results indicates that the method can improve the accuracy of COMPASS single point positioning.

47.2 Improvement of Bancroft Algorithm

47.2.1 Bancroft Algorithm

The Bancroft algorithm based on the Lorentz inner product is mainly in R^4 dimensional space. The Lorentz inner product is defined as: in the R^4 dimensional space, we have

$$\underset{4\times 1}{g} = \left[\underset{3\times 1}{u^T}, \alpha\right]^T, \ \underset{4\times 1}{h} = \left[\underset{3\times 1}{v^T}, \beta\right]^T \tag{47.1}$$

$$< g, h > \ = g^T M h = u^T v - \alpha\beta \tag{47.2}$$

The pseudorange observation between the jth satellite and the user station in GPS pseudorange positioning has such a nonlinear equation written according to the Lorentz inner product

$$\tilde{\rho}^j - b = \sqrt{(x^j - x)^2 + (y^j - y)^2 + (z^j - z)^2} \tag{47.3}$$

Let the position vectors of the GPS satellite be

$$S^j = \begin{bmatrix} x^j & y^j & z^j \end{bmatrix}^T, T^j = \begin{bmatrix} (S^j)^T & \tilde{\rho}^j \end{bmatrix}^T \tag{47.4}$$

and the user position vectors are

$$\mu = \begin{bmatrix} x & y & z \end{bmatrix}^T, X = \begin{bmatrix} \mu^T & b \end{bmatrix}^T \tag{47.5}$$

Then Eq. (47.3) can be expressed according to the Lorentz inner product

$$\frac{1}{2} < T^j, T^j > \; - < T^j, X > \; + \frac{1}{2} < X, X > \; = 0 \tag{47.6}$$

For every pseudorange observation $\tilde{\rho}^j$ and the corresponding satellite, we can list one equation that is similar to Eq. (47.6) without exception. When $d(d \geq 4)$, Eq. (47.6) through d observed satellites and pseudo-range can be expressed as

$$B^T P\theta - B^T PBMX + B^T P\lambda\tau = 0 \tag{47.7}$$

where

$$\underset{d\times 1}{\tau} = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T, \lambda = \frac{1}{2} < X, X >$$

$$\theta = \begin{bmatrix} \theta^1 & \theta^2 & \cdots & \theta^d \end{bmatrix}, \theta^j = \frac{1}{2} < T^j, T^j >$$

$$B = \begin{bmatrix} x^1 & y^1 & z^1 & \tilde{\rho}^1 \\ x^2 & y^2 & z^2 & \tilde{\rho}^2 \\ \vdots & \vdots & \vdots & \vdots \\ x^d & y^d & z^d & \tilde{\rho}^d \end{bmatrix}$$

Then unknown parameters can be solved by Eq. (47.7)

$$\hat{X} = \begin{bmatrix} \hat{\mu} \\ \hat{b} \end{bmatrix} = MF(\lambda\tau + \theta) \tag{47.8}$$

where $F = (B^T PB)^{-1} B^T P$ and λ is still the unknown parameter, so it must be firstly solved.

From Bancroft algorithm, we have

$$< F\tau, F\tau > \lambda^2 + 2(< F\tau, F\theta > \; - 1)\lambda + < F\theta, F\theta > \; = 0 \tag{47.9}$$

We can get two solutions λ_1 and λ_2 by solving Eq. (47.9). Then substitute λ_1, λ_2 into Eq. (47.8), and get a group of solutions corresponding with pseudo-range that are consistent with the original pseudorange measurements.

47.2.2 Weighting Algorithm

From Ref. [2], we can see that Eq. (47.9) is a form of global nonlinear LS solution with the Lorentz norm

$$\sum_{i=1}^{n} < T^j - X, T^j - X >^{2} = \min \tag{47.10}$$

Then

$$V = < T^j - X,\, T^j - X > \; = \; < T^j,\, T^j > \; - 2 < T^j,\, X > \; + \; < X,\, X > \tag{47.11}$$

We can substitute B, θ, τ into Eq. (47.11)

$$V = \theta + \lambda\tau - BM\hat{X} \tag{47.12}$$

The weighting algorithm is based on prior information of pseudorange in tradition method which is rigorous in linear LS solution [10]. But when we write GPS pseudorange equation with Lorentz inner product, we need to change weighting algorithm based on Lorentz inner product.

According to Eq. (47.12), we can treat θ as virtual observation

$$\theta^j = \frac{1}{2} < T^j,\, T^j > \; = \frac{1}{2}\left[(S^j)^T S^j - \tilde{\rho}^j \tilde{\rho}^j\right] \tag{47.13}$$

In consideration of error propagation law, we can get the weight matrix of Lorentz inner product

$$P_L^j = \frac{\delta_0^2}{D(\tilde{\rho}^j)} \frac{1}{\tilde{\rho}^j \tilde{\rho}^j} \tag{47.14}$$

where δ_0^2 is mean square error, $D(\tilde{\rho}^j)$ is variance.

From the model based on elevation cut-off angles, the weight of j satellite can be expressed as

$$p_E^j = \frac{\delta_0^2}{D(\tilde{\rho}^j)} = \sin^2(e) \tag{47.15}$$

Then the weight matrix of Lorentz inner product

$$P_L^j = \frac{\sin^2(e^j)}{\tilde{\rho}^j \tilde{\rho}^j} \tag{47.16}$$

Compare Eq. (47.16) with Eq. (47.15), we can see that the new weighting algorithm takes the distance into consideration which is more accurate when satellites are at different heights.

47.3 Numerical Examples

We process the two-frequency COMPASS static observe data of four areas (urumqi, kashi, kunming, changchun) which was observed from October 7 to October 21 in 2012 year, interval is 1 s. Station coordinates are already known.

In data processing the following four schemes are adopted:

Scheme 1: least squares solution using ionosphere-free linear combination of two-frequency pseudorange, each observation weighted based on its satellite elevation angle;

Scheme 2: least squares solution using the original pseudorange observation, each observation weighted based on its satellite elevation angle;

Scheme 3: Bancroft solution using the original pseudorange observation, each observation weighted based on its satellite elevation angle;

Scheme 4: Bancroft solution using the original pseudorange observation, each observation weighted based on the method in this paper.

In order to see the results clearly, we choose data of urumqi station on October 7, 2012 to analysis. The result of other station and time are similar.

Based on Fig. 47.1 and Table 47.1, we can draw a conclusion that:

1. Compare scheme 1 with other schemes, we can see that the ionosphere-free linear combination can not improve SPP accuracy in COMPASS.
2. Bancroft algorithm improve the accuracy slightly than least square algorithm when both of their observations are weighted based on satellite elevation angle, mainly because Bancroft solution is global nonlinear least squares solution and the iterative least square are susceptible to the influence of its approximation.
3. Compared with other schemes, scheme 4 get the best result which shows that new weighting algorithm based on Lorentz inner product is more accurate and

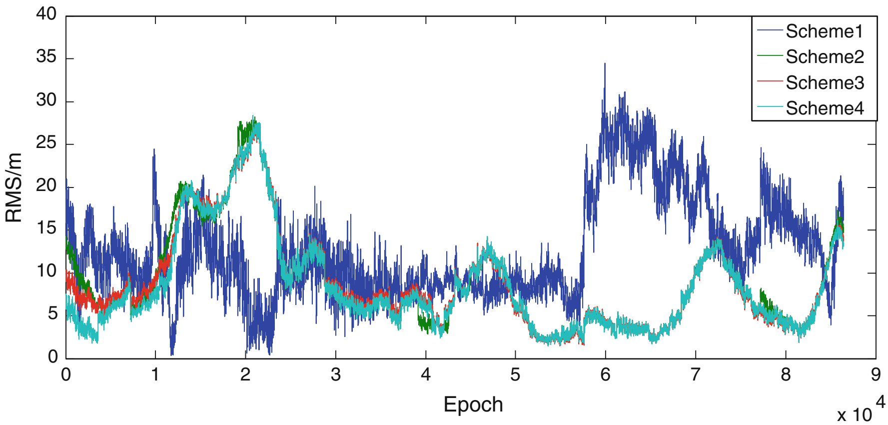

Fig. 47.1 RMS of four schemes

Table 47.1 N, E, U bias of four schemes/m

方案	N	E	U	RMS
Scheme 1	4.65613	4.81255	11.25848	13.09937
Scheme 2	2.10627	1.46580	10.18285	10.50121
Scheme 3	2.15792	1.49941	10.08553	10.42223
Scheme 4	2.03678	1.44668	9.87218	10.18339

Table 47.2 Satellites in epoch 1

PRN	Type	Altitude/m	Elevation angle/°	Weight ratio
1	GEO	35,770,412	44.93437	0.09527
3	GEO	35,779,901	42.12005	0.38142
5	GEO	35,793,493	30.47598	0.35802
7	IGSO	35,706,817	62.88308	0.25868
8	IGSO	35,853,064	58.11976	0.50603
10	IGSO	35,720,130	28.91402	0.48030
11	MEO	21,598,858	44.95042	0.61723
12	MEO	21,589,266	11.85839	1.00000

Table 47.3 N, E, U bias of four schemes in first 10,000 epochs

Scheme	N	E	U	RMS
Scheme 1	3.40756	2.69697	12.80179	13.51927
Scheme 2	1.40868	1.18814	7.46394	7.68807
Scheme 3	1.65410	1.22378	7.75057	8.01904
Scheme 4	0.78356	0.82774	6.15850	6.26309

rigorous than the weighting algorithm based on satellite elevation angle in Bancroft algorithm.

Through the analysis of result, we can be see Bancroft improve the accuracy of COMPASS single point positioning, but the result was not obvious. In order to further analysis, we selected first 10,000 epochs data to be analysis again. In this period, eight satellites have been observed and the satellite information of epoch 1 is listed in Table 47.2. The weight of every satellite observation is compared to the weight of 12th satellite observation.

Based on Tables 47.2 and 47.3, we can draw a conclusion that:

1. GEO satellites are about same altitude with IGSO satellites and higher than MEO satellites.
2. When GEO, IGSO and MEO satellites are observed at the same time, improved Bancroft algorithm improves the accuracy obviously. This is mainly because that MEO satellites are lower than other satellites and the new weighting algorithm based on Lorentz inner product takes the distance into consideration.
3. As MEO satellite increases, the constellation will be more complex, improved Bancroft will get higher accuracy.

47.4 Conclusion

Global nonlinear least squares algorithm (Bancroft algorithm) is applied to the COMPASS single point positioning in this paper. In consideration of the different altitude of satellites, a new weighting algorithm is introduced base on the Lorentz inner product. A lot of observation data is computed to demonstrate the ideas involved. The results indicates that the method can improve the accuracy of COMPASS single point positioning.

Along with development of COMPASS and MEO satellite increases, the new algorithm will be more and more useful.

References

1. Tevfik M (2004) The stochastic modeling of GPS observations. Tuikish J Eng Env Sci 28:223–231
2. Ward P (1996) GPS satellite signal characteristics. In: Understanding GPS principles and applications. Artech House Publishers, pp 83–117
3. Rothaceher M (1997) Processing strategies for regional GPS networks. In: Proceedings of the IAU General Assembly
4. China Satellite Navigation Office (2011) BeiDou navigation satellite system signal in space interface control document
5. Xu G (2007) GPS theory, algorithms and applications, 2nd edn. Springer, Berlin, pp 39–40
6. Bancroft S (1985) An algebraic solution of the GPS equation. IEEE Trans Aerosp Electron Syst AES-21:56–59
7. Zhang Q (2002) Research on nonlinear least square theory and its applications in GPS positioning. Wuhan University, pp 138–144
8. Zhang Q et al (2005) Nonlinear filter method of GPS dynamic positioning based on Bancroft algorithm. Transactions of Nanjing University of Aeronautics & Astronautics, June 2005, pp 170–176
9. Yang Y et al (2005) Analyses and comparisons of some strategies for nonlinear Kalman filter in GPS navigation. Eng Surveying Mapp 14(3):4–7
10. Rengui R et al (2008) Improvement of Bancroft algorithm. J Geodesy Geodyn 28(5):87–90